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Ontario

What Works? Research into Practice

A research-into-practice series produced by a partnership between The Literacy and Numeracy Secretariat and the Ontario Association of Deans of Education.

Research Monograph # 1

can teachers support meaningful,
-quality student interaction in the
 classroom?

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Student Interaction in the Math Classroom: Stealing Ideas or Building Understanding

By Dr. Catherine D. Bruce
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"It was the third math class of the year. My Grade 7 students were unusually eager. We were looking for patterns in a strategic list of solutions generated from a number game. As one student described a complex pattern in the sequence, a second student shouted: 'She stole my idea!' At that point, I knew my work was cut out for me. How could I possibly move this group of competitive students from believing that math was an individual sport where power lies in the hoarding of information and 'getting the answer first', to understanding the exponential power of mathematical thinking when it is shared and built collectively?"

Excerpted from a teacher's journal

Research tells us that student interaction – through classroom discussion and other forms of interactive participation – is foundational to deep understanding and related student achievement. But implementing discussion in the mathematics classroom has been found to be challenging.

The Value of Student Interaction

In the math reform literature, learning math is viewed as a social endeavour.[1,2] In this model, the math classroom functions as a community where thinking, talking, agreeing, and disagreeing are encouraged. The teacher provides students with powerful math problems to solve together and students are expected to justify and explain their solutions. The primary goal is to extend one's own thinking as well as that of others.[3]

Powerful problems are problems that allow for a range of solutions, or a range of problem-solving strategies. Math problems are powerful when they take students beyond the singular goal of computational mastery into more complex math thinking. Research has firmly established that higher-order questions are correlated with increased student achievement, particularly for conceptual understanding.[1] The benefits increase further when students share their reasoning with one another. Reform-based practices that emphasize student

Challenges that Teachers Face

- complexities of teaching mathematics in ways they did not experience as students
- discomfort with their own mathematics knowledge
- lack of sustained professional development opportunities
- greater requirement for faciltiation skills and attention to classroom dynamics
- lack of time, especially in face of curricular demands

interaction improve both problem-solving and conceptual understanding[5,6] without the loss of computational mastery.[7,8] Why then does the traditional mathematics teaching model, focused on basic computational procedures with little facilitation of student discourse, continue to be the common instructional approach in many elementary schools?

Challenges that Teachers Face in Engaging Students

Math teachers face a number of challenges in facilitating high-quality student interaction, or "math-talk". The biggest is the complexity of trying to teach mathematics in ways they did not experience as students.[9,10] Discomfort for some with their own level of math content knowledge[11] and lack of sustained professional development opportunities also make teachers reluctant to adopt math-talk strategies.

Further, the complex negotiation of math-talk in the classroom requires facilitation skills and heightened attention to classroom dynamics. The teacher must model math-talk so that students understand the norms of interaction in the math classroom,[12] encourage students to justify their solutions and build on one another's ideas,[3] and finally step aside as students take increasing responsibility for sustaining and enriching interactions.

Time is another challenge. In the face of curricular demands, the time required for facilitated interaction has been identified by teachers as an inhibitor to implementing math-talk.[13] However, the research also tells us that despite these challenges, teachers have devised some particularly effective strategies for facilitating math-talk.

The Teacher's Role

In an extensive study examining math classroom activity, student interaction was one of ten essential characteristics of effective mathematics teaching.[19] However, left to their own devices, students will not necessarily engage in high-quality math-talk. The teacher plays an important role. According to this same study, three main activities of Ontario teachers who successfully facilitated math-talk were :

1. The teacher assigned tasks that required students to work together to develop joint solutions and problem-solving strategies.
2. The teacher provided instruction on and modeled expected behaviours focusing on group skills, shared leadership, and effective math communication.
3. The teacher urged students to explain and compare their solutions and solution strategies with peers. Students were encouraged to be both supportive and challenging with peers.

Other research[15] has identified two more important roles:

4. The teacher knew when to intervene and when to let the conversation continue even if it was erroneous.
5. Students were evaluated on their math-talk.

Implications for Educational Practice

Five Strategies for Encouraging High-Quality Student Interaction

1. *The use of rich math tasks*

The quality of math tasks is of primary importance. When a task has multiple solutions and/or permits multiple solution strategies, students have increased opportunities to explain and justify their reasoning. If a task involves a simple operation and single solution, there will be little or no opportunity to engage students.

Justification of solutions

Encouraging productive argumentation and justification in class discussions leads to greater student understanding. In a study of four teachers using the same lesson, Kazemi and Stipek[16] found that there were significant differences in the quality of math-talk from class to class. Two of the four classes demonstrated evidence of deeper mathematical inquiry. In these two classes, the teachers explicitly asked students to justify their strategies mathematically and not merely recount procedures.

3. Students questioning one another

Getting students to ask each other good questions is a very powerful strategy. For example, King[17] found that giving students prompt cards, with a range of higher-order questions, led to greater student achievement. The prompts were question stems such as "how are ... and ... similar?" Students applied current content to the questions (e.g., "how are squares and parallelograms similar?"). The students retained more when they used prompt cards than when they spent the same amount of time discussing content in small groups without prompts.

4. Use of wait time

Asking questions that call for higher-level thinking is not particularly helpful if students are not also given sufficient time to do the related thinking. Those teachers who increase the amount of time they give students to respond, allowing even three seconds instead of the usual one, have found that students give more detailed answers expressed with greater confidence. With increased wait time, combined with higher-level questions, student attitudes towards learning improve.[18]

5. Use of guidelines for math-talk

In a district-wide Grade 6 study, teachers were provided with professional development (PD) in mathematics content and pedagogical models for facilitating student interaction.[19] The results on EQAO mathematics assessments, in year-over-year comparisons before and after the PD opportunity, indicated a substantial increase in student achievement, while the reading and writing scores remained consistent. In this project, guidelines for whole-class math-talk were modeled with teachers in active PD sessions and were subsequently implemented by participating teachers. A year later, some teachers were observed using the guidelines, which were still posted in their classrooms. These guidelines (see sidebar) help teachers and students engage in high-quality interaction leading to richer mathematical thinking, and deeper understanding of concepts and related applications.

In sum ...

Let's return to the concern raised in the opening vignette, where shared or similar solutions and strategies are described as the "stealing" of ideas. In order to move beyond this competitive and isolating approach which has had limited success, students must be encouraged to work, think, and talk together while engaging in powerful mathematics tasks. Clearly, the teacher plays a pivotal role in shaping the learning environment. By providing students with a framework for interaction, students can be guided effectively towards working as a learning community in which sharing math power extends understanding and leads to higher levels of achievement.

Guidelines for Whole-Class Math-Talk

1. **Explain: "This is my solution/strategy ..." "I think _____ is saying that ..."**
   - Explain your thinking and show your thinking.
   - Rephrase what another student has said.
2. **Agree with reason: "I agree because ..."**
   - Agree with another student and describe your reason for agreeing.
   - Agree with another student and provide an alternate explanation.
3. **Disagree with reason: "I disagree because ..."**
   - Disagree with another student and explain or show how your thinking/ solution differs.
4. **Build on: "I would like to build on that idea..."**
   - Build on the thinking of another student through explanation, example, or demonstration.
5. **Go beyond: "This makes me think about ..." "Another way to think about this is ..."**
   - Extend the ideas of other students by generalizing or linking the idea to another concept.
6. **Wait time:**
   - Wait to think about what is being said after someone speaks (try five seconds).

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Sharing Best Practice

The Literacy and Numeracy Secretariat has developed a professional learning series to help ·room teachers enhance their mathematical ..owledge and understanding:

- Numeracy Professional Learning Series Regional workshops in January 2007
- Webcast on Mathematical Knowledge for Teaching with Dr. Deborah Loewenberg Ball www.curriculum.org

The Ministry of Education has developed some new resources to share research on teaching and learning and on best practices in education, including:

- Annual Ontario Education Research Symposium
- *Inspire: The Journal of Literacy and Numeracy for Ontario* www.inspirelearning.ca
- *Unlocking Potential for Learning: Effective District-Wide Strategies to Raise Student Achievement in Literacy and Numeracy*
- *What Works? Research into Practice*

For more information: info@ontario.ca

Figure 1. "Buggy" Errors in Children's Double-Digit Addition and Subtraction Adapted from Nagel & Swingen, 1998, p. 167

Computation	Child's Solution
28 +29 471	Alena worked from the left side of the numbers. She added 2 plus 2 and placed 4 under these numbers. She then added 8 plus 9 and wrote 71, reversing the 17 under this column, coming to an answer of 471.
76 -29 53	Andy said, "I took away 2 from the 7, then got 5. Then I took 6 from the 9 and got 3." His answer was 53, which resulted from looking at the ones column and finding the smaller number to take away, regardless of number placement.
76 511 -29 -29 32	Kelly thought that she could not subtract 9 from 6 so she changed the 7 to a 5, "borrowed" 11 (two 1s) from the 10s column and placed it in the units' column, and then subtracted.

Children have difficulty making sense of our traditional North American algorithms for good reason. These algorithms were developed over time to maximize efficiency and accuracy before the time of calculators.[5] They were not meant to maintain sense-making for the learner; instead, they embody many shortcuts based upon extensive mathematics – mathematics often beyond the capacity of the average Grade 2 student. Therefore, there has been an important shift to improve understanding by beginning instruction using children's initial understandings.

Children's Solution Strategies

More than two decades ago Tom Carpenter and his research team[6] began asking children to solve problems without the benefit of direct instruction of methods. They found that children would generate a variety of solution strategies when given, for example, a primary division problem such as this: *Maria's mom baked 42 cupcakes. She is placing them in 7 tins. If she puts the same number in each how many should she place in a tin?*

- At first, most children will model the problem directly by counting out 42 items or drawing 42 cupcakes, then drawing 7 tins and "doling" out the cupcakes into the tins one at a time until they run out. Alternatively, they may count out or draw groups of 7 cupcakes, adding more groups of 7 as needed until they get to 42, and then recounting the groups to determine the number in each tin. At this stage they are concretely modelling the action of the problem.

- Children will learn to replace these direct or concrete modelling procedures with counting strategies, often skip-counting and keeping track on their fingers how many times they counted. Or they may instead add in some fashion, perhaps doubling (7 + 7 = 14), then doubling again (14 + 14 = 28), and then adding (14 + 28 = 42). Children often find doubling easier than other forms of addition.

- Later, they may use derived facts to solve the problem. That is, as they construct some multiplication facts they know, such as fives (which are easier), they use this to derive ? x 7 = 42. If you know that 5 x 7 = 35, then one more 7 will work. Eventually, most children will solve this as a division fact (42 ÷ 7 = 6).

There are many other long-term research projects with similar findings, such as Karen Fuson's Supporting Ten-Structured Thinking Projects[7] and Constance Kamii's ongoing work in Children Reinventing Arithmetic.[8,9] Students in these classrooms have a significantly deeper understanding and enjoyment of the mathematics than their counterparts in traditional instruction classrooms.

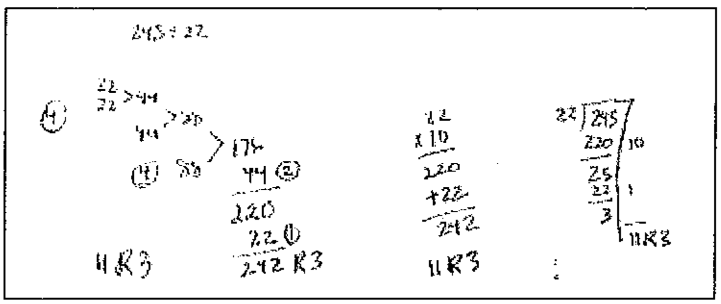
These ideas are now being extended into the junior grades.[10,11] Students who develop a strong foundation in solving problems using their own methods at the primary level can use this knowledge to tackle more challenging problems in the junior grades. While we have somewhat less research on the effectiveness of encouraging student-generated or alternative algorithms at the junior level than the primary, there is mounting evidence that this approach continues to promote deeper understanding and fewer misconceptions or errors than is the case with direct instruction of standard algorithms.[12,13] Moreover there is evidence that these methods are more accessible for all – including students struggling with their mathematics.[14]

What Teachers Can Do to Support Mathematics Learning

It is important to note that this progression of student strategies from early concrete modelling through to efficient, alternative or standard algorithms is neither linear nor developmental. Instead the progression is experiential – the result of classroom experiences in which teachers effectively support children in solving problems using their own methods. How do teachers do this?

At the junior level, for example, it would mean introducing division with an accessible problem, set in a familiar context, rather than as a series of steps to be learned.

Figure 2. Student-Generated Methods and Alternative Algorithm for 245 ÷ 22



- **Pose a problem** such as: *I have a large bag of 245 M&Ms. If we divide this evenly among the class (22 students), how many would each of you get?*

- **Pair students with a partner who is at the same mathematical level** in order to encourage full participation of both students.

- **Allow students to try to solve the problem with the method that makes sense to them.** Students may add up, multiply, subtract, or divide to solve this problem (see the first two examples in Figure 2 for typical solutions). Students will likely require a full period to solve their problem and get ready to share their ideas during the math discussion or "congress" at a later time. Choose a few pairs to share their thinking with the class.

- **Introduce a math congress to focus student thinking on one or two strategies or perhaps "Big Ideas."** For example, when students solve a division problem (such as the M&M problem) using different strategies it is an opportunity to ask: Why is it we can multiply or divide to find the same answer? The teacher can make use of students' varied solutions to explore the Big Idea that multiplication is the inverse of division.[15] In addition to helping students learn to calculate with greater understanding and capacity, these methods also allow teachers to capitalize on children's thinking in order to deepen their knowledge of mathematics – a capitalization not available when students are restricted to the traditional method or calculator.

Implications for Educational Practice

For More Discussion ...

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For an Ontario Classroom in Action ...

- Miller, L. (2004). Brookmede Public School and École catholique St. Antoine share tips for inspiring young math minds. *Professionally Speaking.* http://www.oct.ca/publications/professionally_speaking/june_2004/math.asp

- Give students many opportunities to solve different division problems so that they will slowly progress towards multiplying up or subtracting greater "chunks" or copies of the divisor. At this time you could introduce the "Dutch"[10] or "accessible division"[12] or "alternative division"[11] algorithm as a way to structure their thinking (see the final solution in Figure 2). If students are already taking away or multiplying and adding up larger multiples of the divisor, this is a structure that will make sense to them and be easily adopted. Finally, students will likely also bring in traditional algorithms that can be explored to examine the mathematics and learn why and how they work.

It must be stated that this is a highly demanding approach requiring mathematical and instructional knowledge, perseverance and patience. Many teachers express doubts about their ability to teach, and their students' ability to learn, mathematics in this fashion.[17] They often find the first few weeks particularly challenging as children, and sometimes their parents, expect direct instruction of algorithms – mathematics as they knew it.[18] The eventual rewards of these instructional changes, however, are classrooms where more children genuinely understand and enjoy mathematics than has been the norm.

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Professional Learning

Literacy and Numeracy Secretariat developed a range of resources to help classroom teachers enhance their mathematical knowledge and understanding:

- **Numeracy Professional Learning Series** on teaching addition and subtraction, multiplication and division, fractions and per cents, and learning through problem solving www.curriculum.org/LNS/coaching
- **Mathematical Knowledge for Teaching**
  Webcast featuring Deborah Loewenberg Ball
  www.curriculum.org/secretariat/november2.html
- **Making Mathematics Accessible to All Students**
  Webcast featuring Mary Lou Kestell, Kathryn Kubota-Zarivnij, and Marian Small
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WHAT WORKS? Research into Practice

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Research Monograph # 22

What kind of mathematics problems help students develop deep, conceptual understanding?

Research Tells Us

- Many students lack a deep understanding of mathematical concepts.
- Classroom teachers find it difficult both to develop a real-life hook for students and to allow students to work through problem solving independently.
- PBL is a promising approach not only to build mathematics understanding but also to test students' conceptual knowledge.
- PBL requires teachers to present students with multifaceted, real-life problems and to act as facilitators supporting students in organizing their own learning.

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Problem-Based Learning in Mathematics

A Tool for Developing Students' Conceptual Knowledge

By Sheryl MacMath, John Wallace, and Xiaohong Chi,
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Mathematics teachers must teach students not only to solve problems but also to learn about mathematics through problem solving.[1] While "many students may develop procedural fluency ... they often lack the deep conceptual understanding necessary to solve new problems or make connections between mathematical ideas."[2] This presents a challenge for teachers: problem-based learning (PBL) provides opportunities for teachers to meet this challenge.

PBL exists as a teaching method grounded in the ideals of constructivism and student-centred learning. When using PBL, teachers help students to focus on solving problems within a real-life context, encouraging them to consider the situation in which the problem exists when trying to find solutions.[3] The majority of research examining PBL focuses on its use in medical schools, with the key features being (a) the use of collaborative small-group work, (b) a student-centred approach, (c) the teacher as facilitator and (d) the use of real-life problems as the organizing focus.[4]

In the medical arena, groups of students are given a set of realistic patient symptoms and expected to research possible diagnoses and courses of treatment; groups work independently, developing and answering their own questions. If, during this diagnostic phase, a group is unsuccessful in addressing key issues, the instructor notes this on their assessment but does not provide the solution.[4] In the classroom setting, it is this aspect of PBL which presents the most significant challenge, requiring teachers to shift from direct instruction to supporting students organize their own learning.[5]

From Medical Model to School Classroom

Studies have shown that teachers may have difficulty *not* directing students, *not* determining student progressions and *not* correcting errors.[6,7] For the PBL approach to work, however, teachers need to take on the role of facilitator, encouraging students to work through each problem; this role is "multifaceted and require[s] flexibility" (p. 209).[8]

When starting units using a PBL model, research suggests that elementary teachers find it difficult to develop an appropriate *hook*[8] – a real-life problem that does not have a single or pre-determined solution and, thus, enables students to develop a variety of answers. In this sense, the value of the problem resides in helping students to develop both an understanding of the mathematics and the ability to apply it.[9]

From the authors' case study ...

"As she walked around the room, Ms. Perry was stunned. She was impressed with students' focus on reasonableness, their rechecking of possible solutions, and their perseverance. However, she was shocked at their inability to solve the problem."

"Although teachers can implement PBL at the beginning of a unit, using a multifaceted problem to create enthusiasm for learning new knowledge and skills, our study of Ms. Perry's class illustrated that PBL can also be used to check for student misconceptions after a unit of study has been completed."

A Case Study

The challenges inherent in developing a multifaceted problem and maintaining the teacher's role as facilitator are exemplified in the authors' case study of an Ontario sixth grade teacher who introduced a real-life premise for a follow-up unit on multiplication and percentages. Inviting her students to think of themselves as managers of a new hockey team, she asked them to solve a range of multifaceted problems, only to learn that they had almost no conception of either multiplication or percentages outside of the context of the traditional math unit.

The Problem

Ms. Perry* posed a multifaceted problem that focused on multiplication and percentages. In groups of five, students received the following instructions: "Your next job is developing an 80 game schedule. From the 80 games, 30% have to be from the same division, 15% from the North East Division, 15% from the South East Division and the remaining 40% from the Western Conference." As multiplication and percentages had been covered in a full unit just two weeks prior, the teacher expected students to quickly calculate the number of home games and move on to looking at travel distances. Instead, all groups were stumped.

Trying to Solve the Problem

Students tried a variety of mathematical procedures to come up with a reasonable answer. They tried dividing: 30 went into 80 twice with 20 left over, but 20 did not make sense. They tried calculating the decimal, but 30/80 = .375 and you can not play part of a game. Students were creative in their choices of operations and demonstrated an understanding of what would be considered a reasonable answer: 30% was a little greater than 25% ... given that 25% was the "same" as dividing by four, students knew that a reasonable answer had to be a little greater than 20 ... but how much greater? Many students, when they came up with a number, would try to check it to see if the same procedure, when used with the other percentages, yielded a total of 80 games. Repeatedly, it did not.

As she walked around the room, Ms. Perry was stunned. She was impressed with students' focus on reasonableness, their rechecking of possible solutions, and their perseverance. However, she was shocked at their inability to solve the problem. She repeatedly commented that students had already been tested and had received a C+ or higher. Interviews with students revealed the root of the problem: Context mattered.

The Importance of Knowing "When"

The unit completed by students prior to this task was representative of most math units. Students worked through a textbook and numerous practice sheets on percentages. They practised pulling information out of the written problems and applying the procedures they had learned to come up with their answers. Everything they had done with respect to percentages fell within the percentage unit; there were specific procedures to follow. Students did not have to decide *when* to use a certain procedure.

* All names are pseudonyms.

When Ms. Perry asked the class, "What is 30% of 80?" a few students replied that *of* meant multiplication. However, 30 times 80 did not make sense. Another student suggested using the decimal (0.3). That yielded an answer of 24. This was a reasonable answer. When students calculated the rest of the percentages using the same procedure and revealed a total of 80, they were glad to have the correct answer and moved on to the next part of the task. However, mini-interviews with students revealed a misconception. When asked why they had not tried multiplying by the decimal earlier, the majority of students replied that "Multiplication always makes a bigger number. We needed a number smaller than 80." This response demonstrated that, although students may have understood that 30% represented less than a whole, they did not have a conceptual understanding of multiplication.

Implications for Classroom Practice

Although teachers can implement PBL at the beginning of a unit, using a multifaceted problem to create enthusiasm for learning new knowledge and skills, our study of Ms. Perry's class illustrated that PBL can also be used to check for student misconceptions after a unit of study has been completed.

Classroom Examples

To design your multifaceted problem, focus on identifying where particular mathematical concepts are used regularly by different individuals in society. Try to link the problem with a variety of school curricula.

For Grades K and 1

- Integrate your math PBL with social studies when you study families.
- Use this activity to observe if students know whether to use addition or subtraction.
- After teaching about families, have students draw a picture of everyone in their family.
- With the numbers 0 to 10 written across the bottom of the chalkboard, have students tape their family picture above the number that represents the number of people in their family.
- Have students work in pairs to compare the size of their families. During this activity, be sure not to use the terms "plus" or "minus." See if students know which operation would be useful.
- Have students change partners and come up with questions regarding the family chart (e.g, "How many more people are in Mike's family than Jenna's?"). Students can then share their questions and solutions with the class.

For Grades 2 and 3

- An understanding of adding, multiplying and estimating is required every time we shop at a grocery store. Design a multifaceted problem around shopping in a grocery store.
- Distribute grocery store flyers to each group.
- Have each group calculate how much money they would need to buy enough food (you could link to the Canadian Food Guide) to feed their group for the day.
- Students must calculate amounts of each food item, cost, tax and final total.
- Make a contest of it: Which group can meet the Canadian Food Guide requirements for the day with the lowest budgeted cost?

For Grades 4 and 5

- Students working on measures of central tendency (mean, mode, median) and visual representations of their data can work far beyond just analyzing test scores – have students create their own surveys.
- Link your math class with social studies. Choose a school-wide, municipal or provincial issue.

Using PBL to diagnose student misconceptions ...

- Choose a curriculum objective that you have already taught during the year.
- Imagine real-life situations in which students could use the knowledge and skills associated with those curriculum objectives.
- Have students work in small groups of three or four.
- Ensure that all group members contribute equally by using group-role assignments (e.g., recording the work, handling materials and monitoring group participation) that rotate every 30 to 60 minutes.
- Plan a number of opportunities for all groups to sit together, share progress reports and present questions or concerns; in this way, peers continue to act as sources of information and assistance.

"Research emphasizes the value of problem-based learning for extending student thinking and creativity."

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- In groups of four or five, have students design a survey that uses a numeric-rating scale for answers.
- During lunch and recess breaks, have students administer the survey to other classrooms.
- Have students calculate their measures of central tendency, using a variety of graphs to represent the responses they gathered.
- Conclude by having students share their findings with the rest of the school, either on bulletin boards, during school-wide presentations or in a mini-news report to be given out to each classroom.

For Grades 6 and 7

- Have students use their knowledge of ratios to design a model ice rink.
- Assign students, in groups of four or five, the task of building a Canadian hockey rink to scale. Students can go on the internet to look up the actual dimensions of the rink.
- Provide materials such as styrofoam, paint, popsicle sticks and glue guns. Monitor groups as they construct their ice rinks.
- Watch for common errors such as only scaling one measurement or confusing the idea of scaling with changing units of measure (e.g., switching from metres to centimetres without realizing that they are dividing by 100).
- Integrate this activity with science and let students practise designing electrical circuits by having them add a working light and buzzer.

In Sum

Research emphasizes the value of PBL for extending student thinking and creativity. Multifaceted problems (those that mimic real-life problems and allow a variety of ways to reach a solution) can also be used in the classroom to reveal student misconceptions that traditional tests miss. Our observations of Ms. Perry's class reveal that there is value in having students demonstrate they know when to use specific procedures by working through problems.

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